

Fig. 1 – Species abundance paradox data, modified from Orłóci (1978). The paradox is that the Euclidean distance between sites 1 and 2, which have no species in common, is smaller than that between sites 1 and 3 which share species 2 and 3; this example shows that the Euclidean distance is not appropriate for species abundance data. With the other coefficients listed below, the distance between sites 1 and 2 is larger than that between sites 1 and 3; furthermore, the distance between sites 1 and 2 is the same as between sites 2 and 3.

Species abundance paradox data
(3 sites, 3 species)

	Species 1	Species 2	Species 3
Site 1	0	1	1
Site 2	1	0	0
Site 3	0	4	8

$$D_{\text{Euclidean}}(\mathbf{x}_1, \mathbf{x}_2) = \sqrt{\sum_{j=1}^p (y_{1j} - y_{2j})^2}$$

$$\mathbf{D} = \begin{bmatrix} 0.0000 & 1.7321 & 7.6158 \\ 1.7321 & 0.0000 & 9.0000 \\ 7.6158 & 9.0000 & 0.0000 \end{bmatrix}$$

$$D_{\text{Bray-Curtis}}(\mathbf{x}_1, \mathbf{x}_2) = \frac{\sum_{j=1}^p |y_{1j} - y_{2j}|}{\sum_{j=1}^p (y_{1j} + y_{2j})} = 1 - \frac{2W}{A+B}$$

$$\mathbf{D} = \begin{bmatrix} 0.0000 & 1.0000 & 0.7143 \\ 1.0000 & 0.0000 & 1.0000 \\ 0.7143 & 1.0000 & 0.0000 \end{bmatrix}$$

$$D_{\text{chord}}(\mathbf{x}_1, \mathbf{x}_2) = \sqrt{\sum_{j=1}^p \left(\frac{y_{1j}}{\sqrt{\sum_{j=1}^p y_{1j}^2}} - \frac{y_{2j}}{\sqrt{\sum_{j=1}^p y_{2j}^2}} \right)^2}$$

$$\mathbf{D} = \begin{bmatrix} 0.0000 & 1.4142 & 0.3204 \\ 1.4142 & 0.0000 & 1.4142 \\ 0.3204 & 1.4142 & 0.0000 \end{bmatrix}$$

$$D_{2_{\text{metric}}}(\mathbf{x}_1, \mathbf{x}_2) = \sqrt{\sum_{j=1}^p \left(\frac{1}{y_{+j}} \frac{y_{1j}}{y_{1+}} - \frac{y_{2j}}{y_{2+}} \right)^2}$$

$$\mathbf{D} = \begin{bmatrix} 0.0000 & 1.0382 & 0.0930 \\ 1.0382 & 0.0000 & 1.0352 \\ 0.0930 & 1.0352 & 0.0000 \end{bmatrix}$$

$$D_{2_{\text{distance}}}(\mathbf{x}_1, \mathbf{x}_2) = \sqrt{\sum_{j=1}^p \left(\frac{1}{y_{+j}/y_{++}} \frac{y_{1j}}{y_{1+}} - \frac{y_{2j}}{y_{2+}} \right)^2}$$

$$\mathbf{D} = \begin{bmatrix} 0.0000 & 4.0208 & 0.3600 \\ 4.0208 & 0.0000 & 4.0092 \\ 0.3600 & 4.0092 & 0.0000 \end{bmatrix}$$

$$D_{\text{Hellinger}}(\mathbf{x}_1, \mathbf{x}_2) = \sqrt{\sum_{j=1}^p \left[\sqrt{\frac{y_{1j}}{y_{1+}}} - \sqrt{\frac{y_{2j}}{y_{2+}}} \right]^2}$$

$$\mathbf{D} = \begin{bmatrix} 0.0000 & 1.4142 & 0.1697 \\ 1.4142 & 0.0000 & 1.4142 \\ 0.1697 & 1.4142 & 0.0000 \end{bmatrix}$$