Origine des structures spatiales en écologie
Conséquences des structures spatiales pour les tests statistiques

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Cours *Biostatistique et écologie numérique*

Ifremer, Brest, 6-10 juin 2005

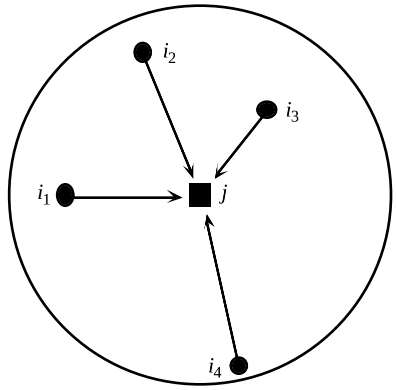


Figure 1.4 The value at site j may be modelled as a weighted sum of the influences of other sites i located within the zone of influence of the process generating the autocorrelation (large circle).

1 — Types of spatial structures

A spatial structure may appear in a variable **y** because the process that has produced the values of **y** is spatial and has generated autocorrelation in the data; or it may be caused by the dependence of **y** upon one or several causal variables **x** which are spatially structured; or both processes. In both cases, spatial correlation will be found when analyzing the data (Chapter 11). The spatially-structured causal variables **x** may be explicitly identified in the model, or not; see Table 13.3.

Autocorrelation

• Model 1: autocorrelation — The value y_j observed at site j on the geographic surface is assumed to be the overall mean of the process (μ_y) plus a weighted sum of the centred values $(y_i - \mu_y)$ at surrounding sites i, plus an independent error term ε_j:

$$y_j = \mu_y + \Sigma f(y_i - \mu_y) + \varepsilon_j \qquad \textbf{(1.1)}$$

The y_i's are the values of **y** at other sites i located within the zone of spatial influence of the process generating the autocorrelation (Fig. 1.4). The influence of neighbouring sites may be given, for instance, by weights w_i which are function of the distance between sites i and j (eq. 13.19); other functions may be used. The total error term is $[\Sigma f(y_i - \mu_y) + \varepsilon_j]$; it contains the autocorrelated component of variation. As written here, the model assumes stationarity (Subsection 13.1.1). Its equivalent in time series analysis is the autoregressive (AR) response model (eq. 12.30).

Induced spatial dependence

• Model 2: spatial dependence — If one can assume that there is no autocorrelation in the variable of interest, the spatial structure may result from the influence of some explanatory variable(s) exhibiting a spatial structure. The model is the following:

$$y_j = \mu_y + f(\text{explanatory variables}) + \varepsilon_j \qquad \textbf{(1.2)}$$

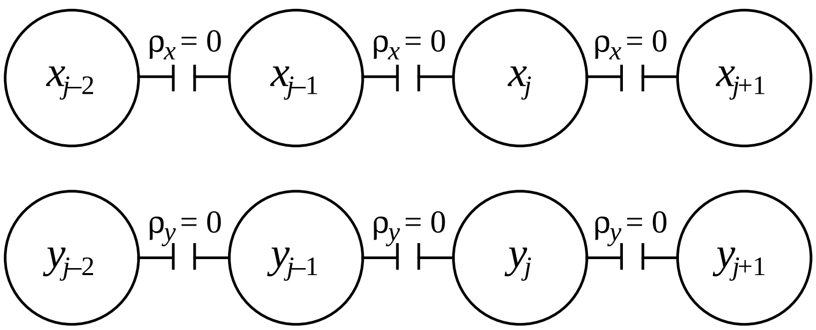
where y_j is the value of the dependent variable at site j and ε_j is an error term whose value is independent from site to site. In such a case, the spatial structure, called

Origin of spatial structures for a variable of interest **y**

Modified from:
Fortin, M.-J. and M. R. T. Dale. 2005. *Spatial analysis – A guide for ecologists.* Cambridge University Press, Cambridge. 380 pp.
and Legendre & Legendre *Numerical ecology* (1998, p. 11)

Consider an explanatory variable **x** and a response variable **y**

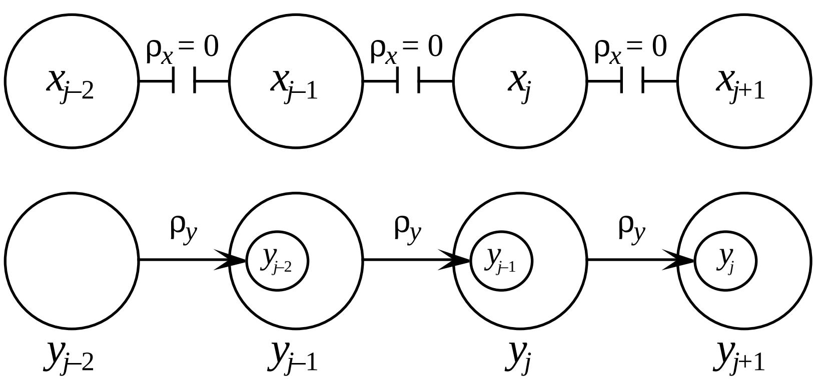
Model 1: No autocorrelation in **x**
No autocorrelation in **y**
y independent of **x** (uncorrelated)



No spatial structure in **y**,
no functional dependence of **y** on **x**

$y_j = \varepsilon_j$

Model 2: No autocorrelation in **x**
Autocorrelated spatial structure in **y**
y independent of **x** (uncorrelated)



Directional autoregressive model

$y_j = \rho_y y_{j-1} + \varepsilon_j$

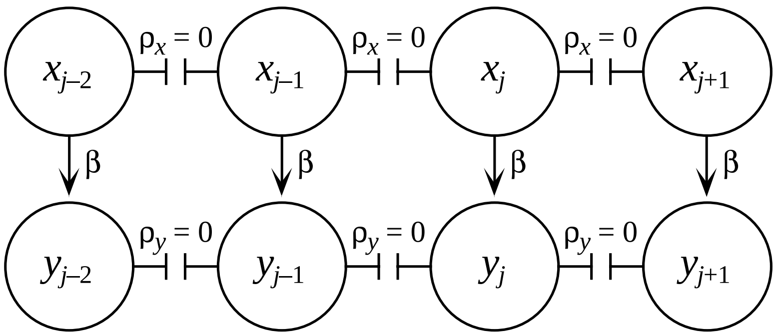
no functional dependence of **y** on **x**

More generally for a 1- or 2-dimensional surface with bi-directional effects: autocorrelation

$y_j = \mu_y + \sum f(y_i - \mu_y) + \varepsilon_j$

where the y_i's are the values of **y** at other sites *i* located within the zone of spatial influence of the process generating autocorrelation in **y** (Fig. 1.4).

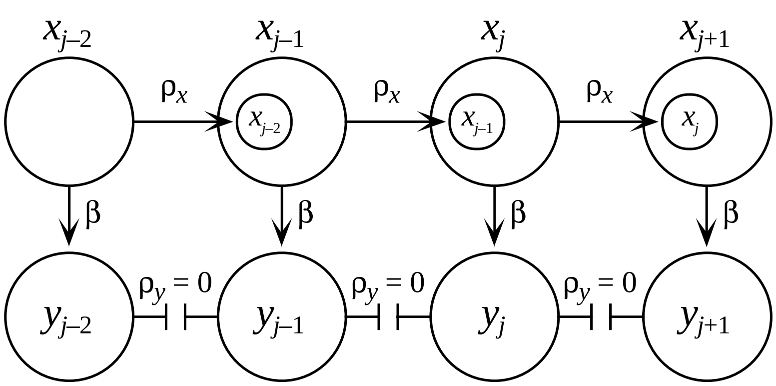
Model 3: No autocorrelation in **x** (centred values)
No autocorrelation in **y**
y depends on **x**



Functional dependence of **y** on **x**

$y_j = \mu_y + \beta x_j + \varepsilon_j$

Model 4: Autocorrelated spatial structure in **x** (centred values)
No autocorrelation in **y**
y depends on **x**

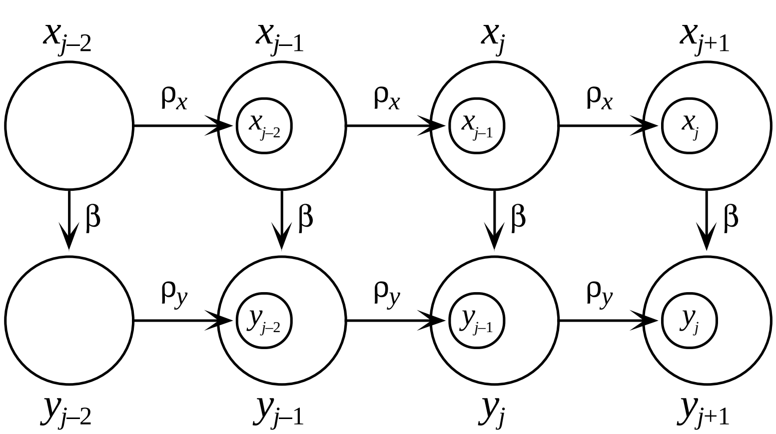


Induced spatial dependence: the spatial structure of **y** results from the dependence of **y** on **x** which is spatially autocorrelated

Directional autoregressive model: $x_j = \rho_x x_{j-1} + \zeta_j$, $y_j = \beta x_j + \varepsilon_j$

More generally for a surface: $y_j = \mu_y + \beta x_j + \varepsilon_j$

Model 5: Autocorrelated spatial structure in **x** (centred values)
Autocorrelated spatial structure in **y**
y depends on **x**

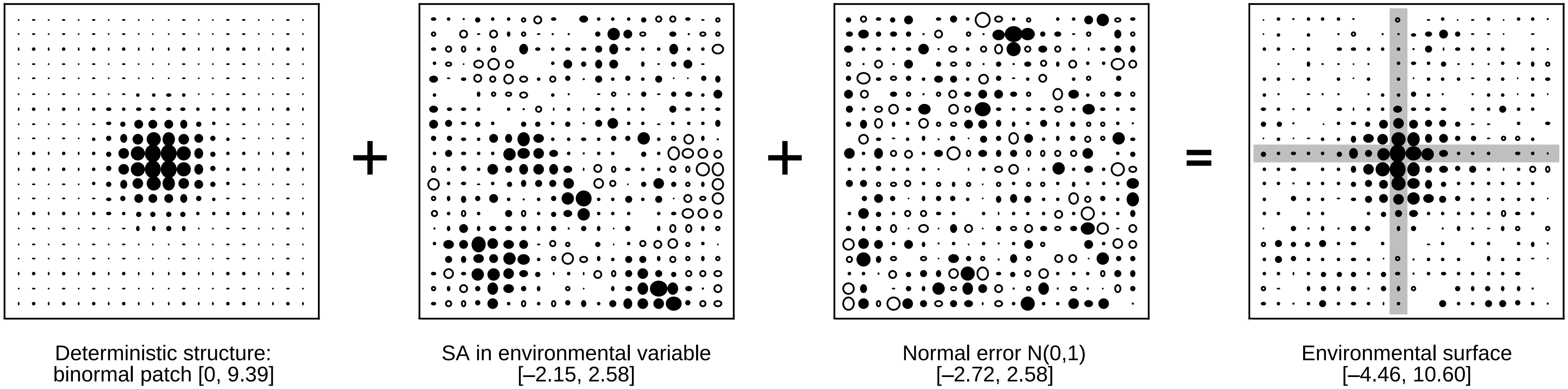


Autocorrelation *plus* induced spatial dependence: the spatial structure of **y** results from autocorrelation in **y** and from the dependence of **y** on **x** which is spatially autocorrelated

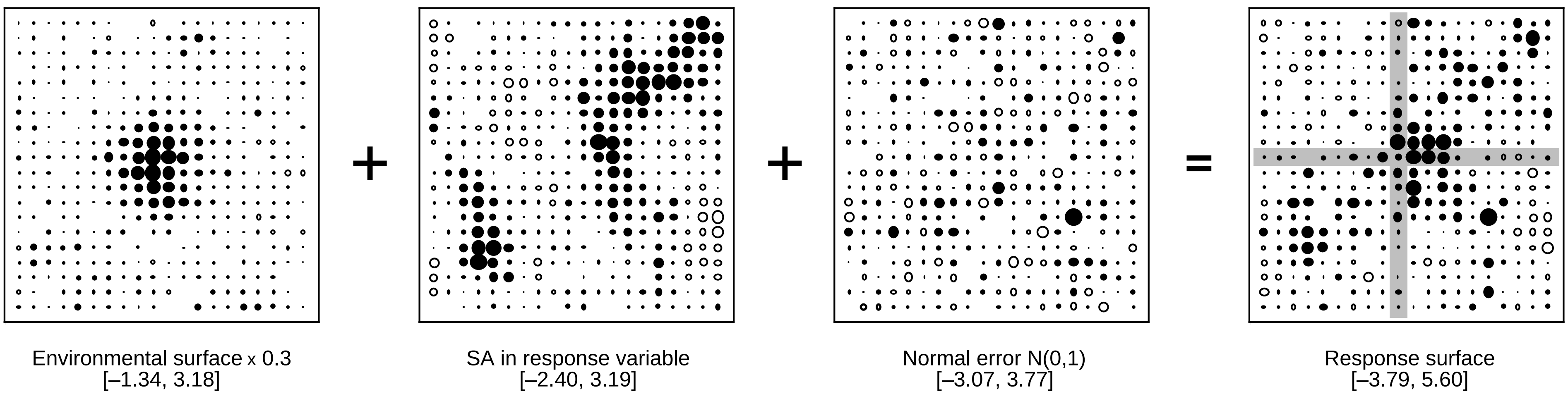
Directional autoregressive model: $x_j = \rho_x x_{j-1} + \zeta_j$, $y_j = \rho_y y_{j-1} + \beta x_j + \varepsilon_j$

More generally for a surface: $y_j = \mu_y + \sum f(y_i - \mu_y) + \beta x_j + \varepsilon_j$

Construction of the environmental surface



Construction of the response surface



<u>Legendre et al. 2002, Fig. 1</u>

Legendre, P., M. R. T. Dale, M.-J. Fortin, J. Gurevitch, M. Hohn and D. Myers. 2002. The consequences of spatial structure for the design and analysis of ecological field surveys. *Ecography* 25: 601-615.

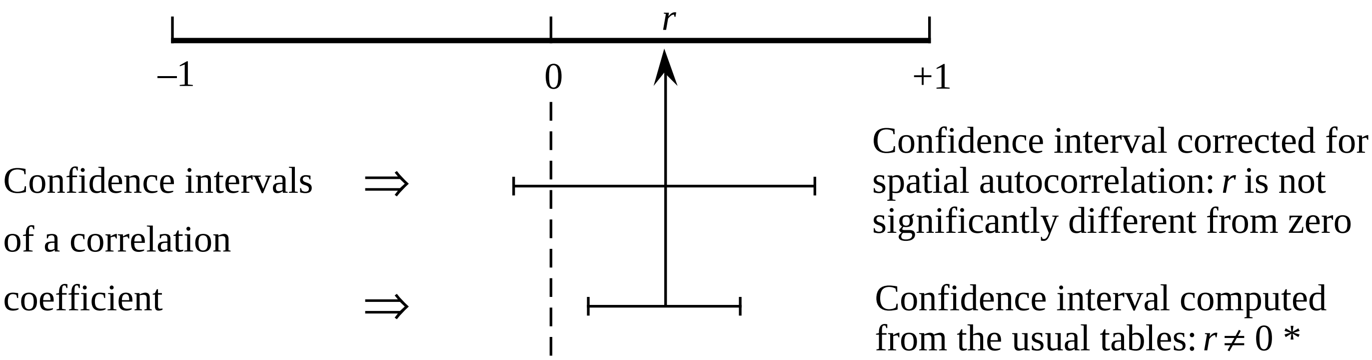
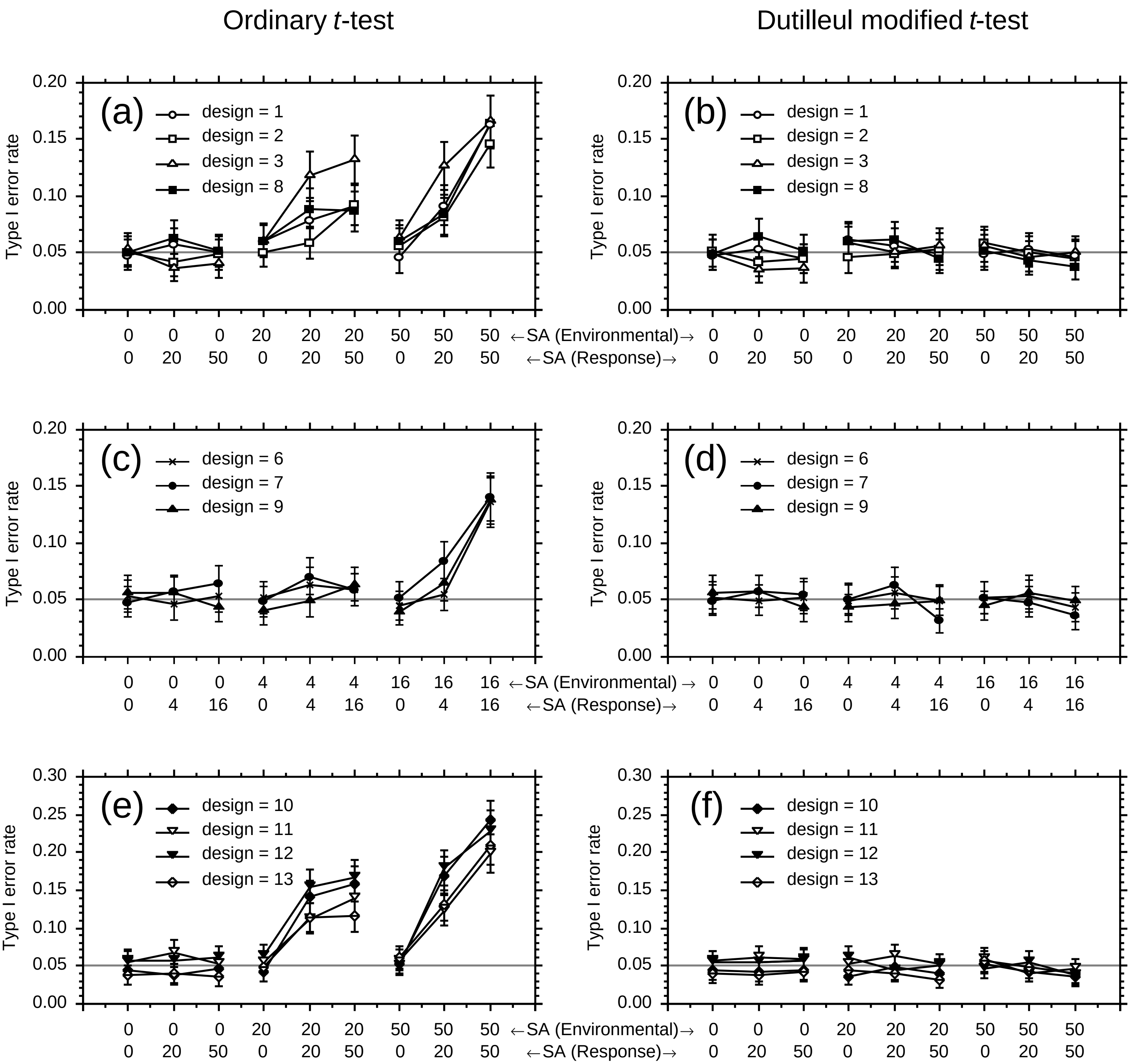


Figure 1.5. Effect of positive spatial autocorrelation on tests of correlation coefficients; * means that the coefficient is declared significantly different from zero in this example.



Legendre et al. 2002, Fig. 2A

Legendre, P., M. R. T. Dale, M.-J. Fortin, J. Gurevitch, M. Hohn and D. Myers. 2002. The consequences of spatial structure for the design and analysis of ecological field surveys. *Ecography* 25: 601-615.

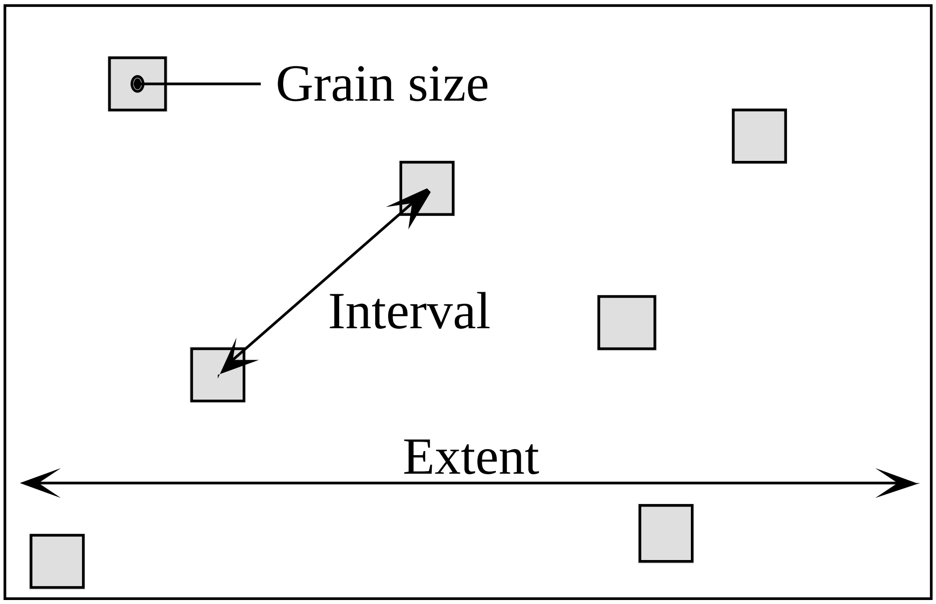
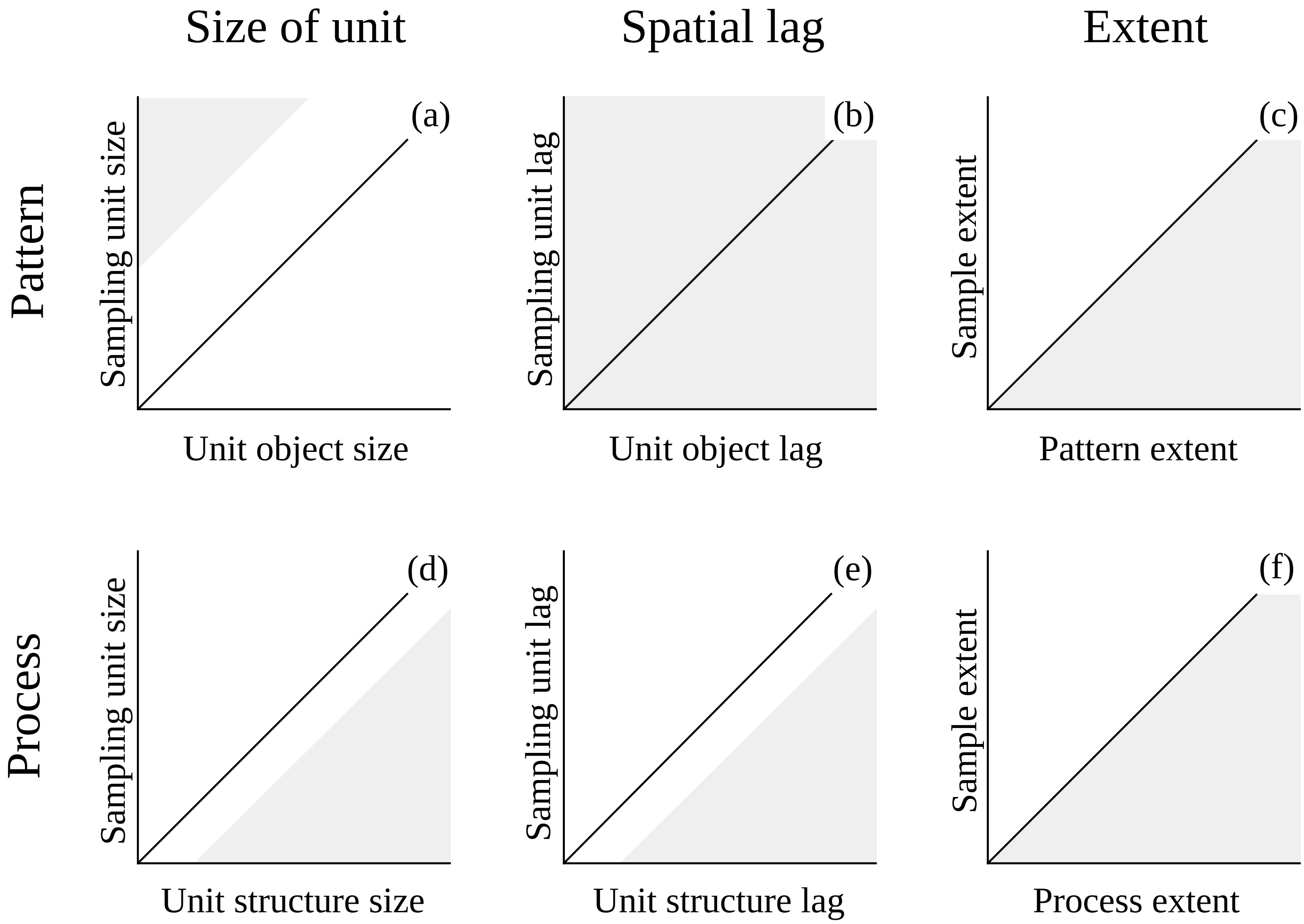
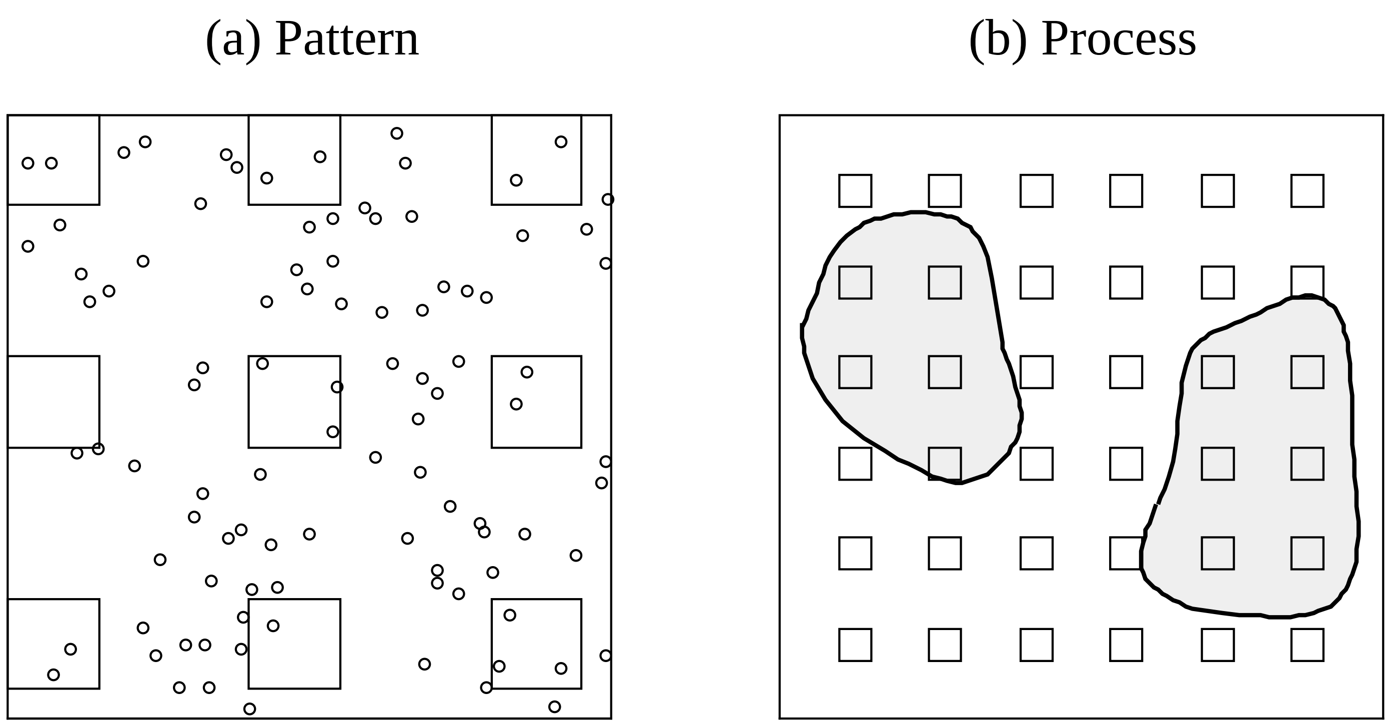


Figure 13.1 Components of a sampling design are the grain size, sampling interval, and extent. In the Figure, the sampling units are represented by squares.





Figures 10 and 11 from:

Dungan, J. L., J. N. Perry, M. R. T. Dale, P. Legendre, S. Citron-Pousty, M.-J. Fortin, A. Jakomulska, M. Miriti and M. S. Rosenberg. 2002. A balanced view of scaling in spatial statistical analysis. *Ecography* 25: 626-640.