

The dynamics of spatially-extended populations

Statistics and theory for spatial correlation functions

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Spatial autocorrelation (e.g. Fortin & Legendre 1989, Legendre 1993) (= spatial ‘self-similarity’)

Variable N measured at locations i, j :

Spatial autocorrelation: $\rho_{ij} = (N_i - \bar{N})(N_j - \bar{N}) / \sigma_N^2$

‘Classical’ statistical inference → *not correct*

Theoretical ecology: → *critical quantity* ✓

Ecological processes leave detailed signatures in the spatial autocorrelation and cross-correlation functions

Part 1) Theory & Moment equations

Simplest case - single species & dispersal (Spatial 'BIDE')

Changes in autocorrelation in locations $\{i, j\}$

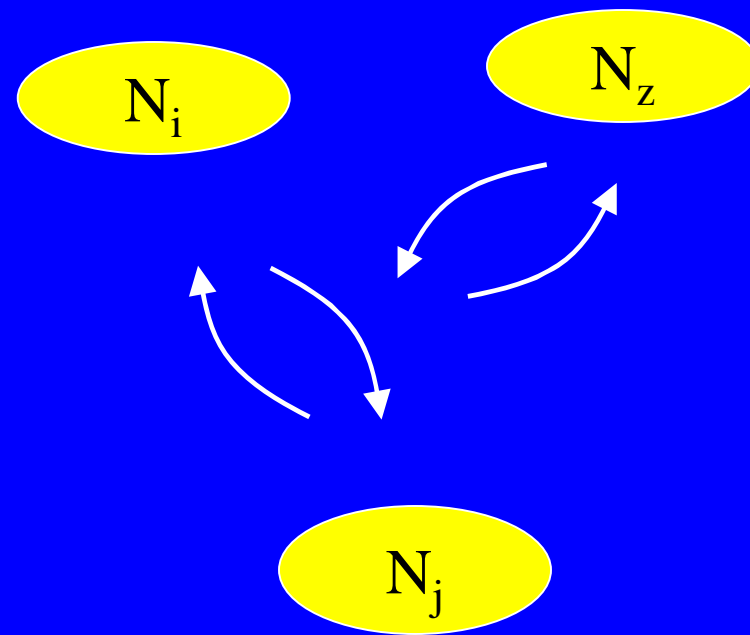
Depends on:

- Birth
- Death

- migration between i and j

- emigration from i *not to* j
from j *not to* i

- immigration from *not-j* to i
from *not-i* to j



Moment equations for dynamics and dispersal:

$$\frac{dN_i}{dt} = \underbrace{N_i R(N_i)}_{\text{Local pop dyn}} - \underbrace{pN_i + p \int K(d_{iz}) N_z dz}_{\text{Movement}} + \underbrace{\sigma_R \frac{dB_i}{dt}}_{\text{Noise}}$$

$$\frac{dC(d)}{dt} = 2R(\bar{N}) + 2\bar{N}R'(\bar{N})C(d) - pC(d) + p(K * C)(d) + \sigma_R^2 \delta(d)$$

$C(d)$ Covariance at distance d

$K(d)$ Dispersal kernel

N_i Number at i

σ_R Magnitude of environmental stochasticity

$R(N)$ Density-dependent growth

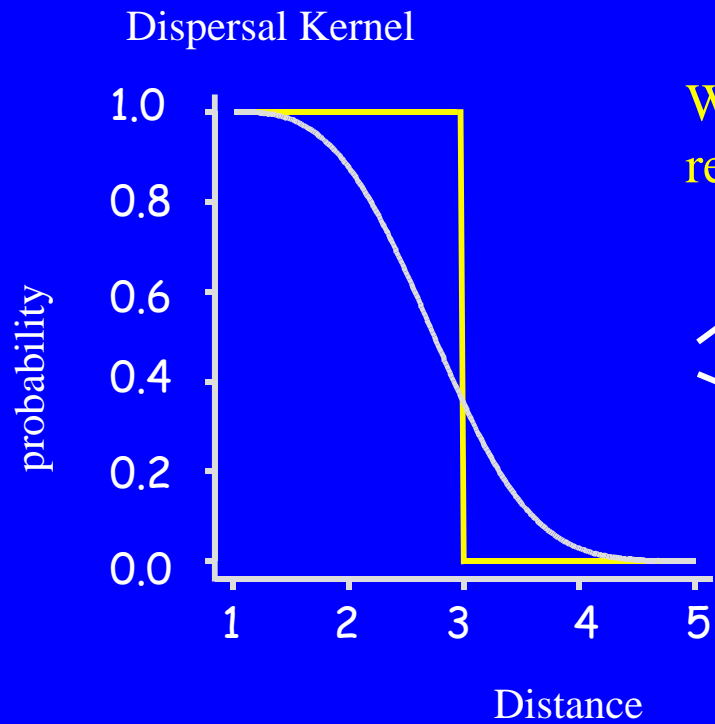
$\delta(d)$ Environmental correlation function

p Dispersal probability

$K * C$ Convolution of $K(d)$ and $C(d)$

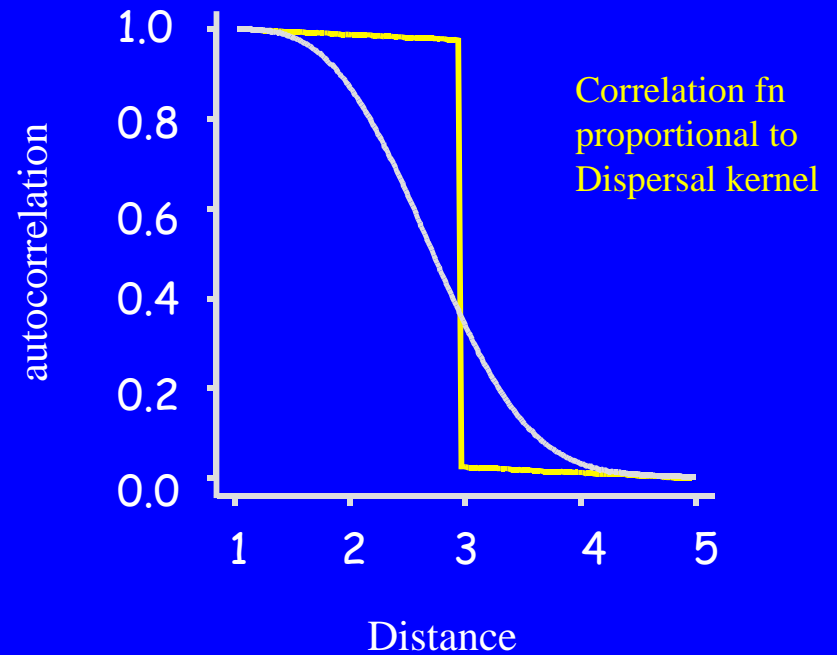
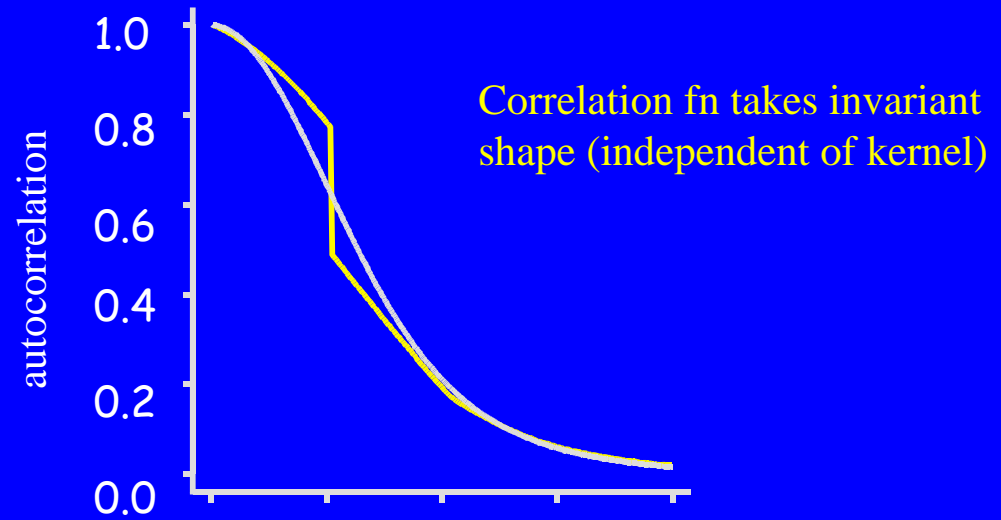
$\frac{dB_i}{dt}$ Derivative of Brownian motion (cf stochastic ODE's)

Dispersal induced autocorrelation:



Weak regulation

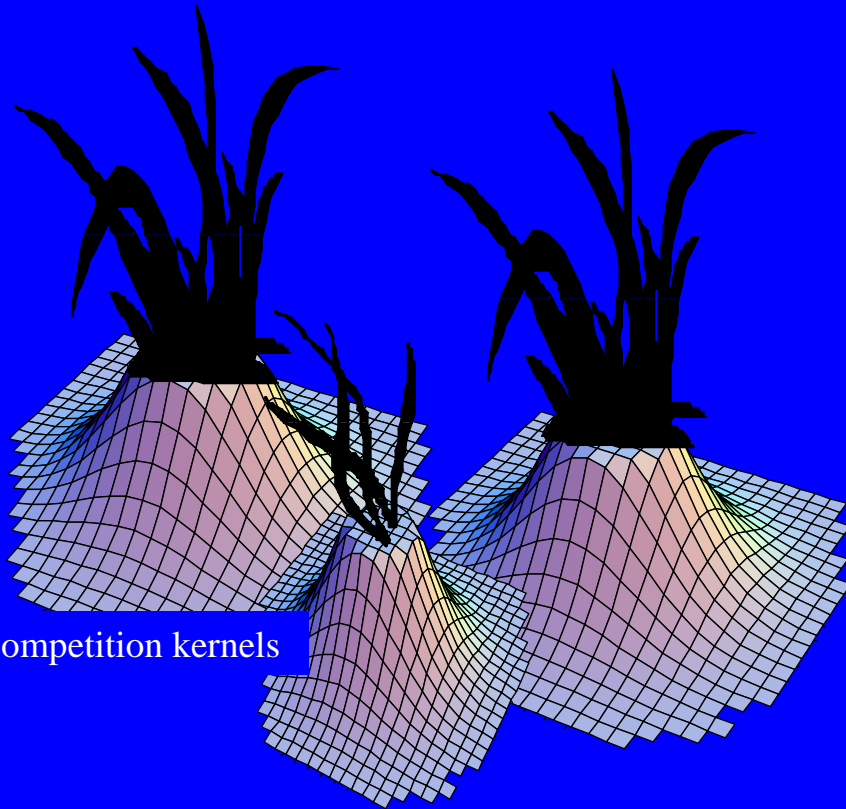
Strong regulation



- Gaussian
- Tophat

What about interspecific processes: (1) spatial competition

1.



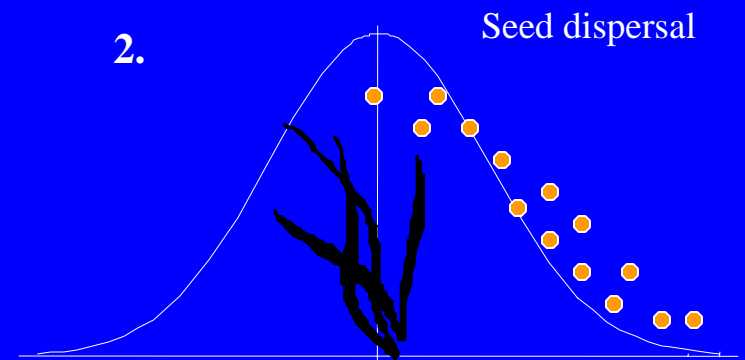
Competition kernels



Two spatial processes:

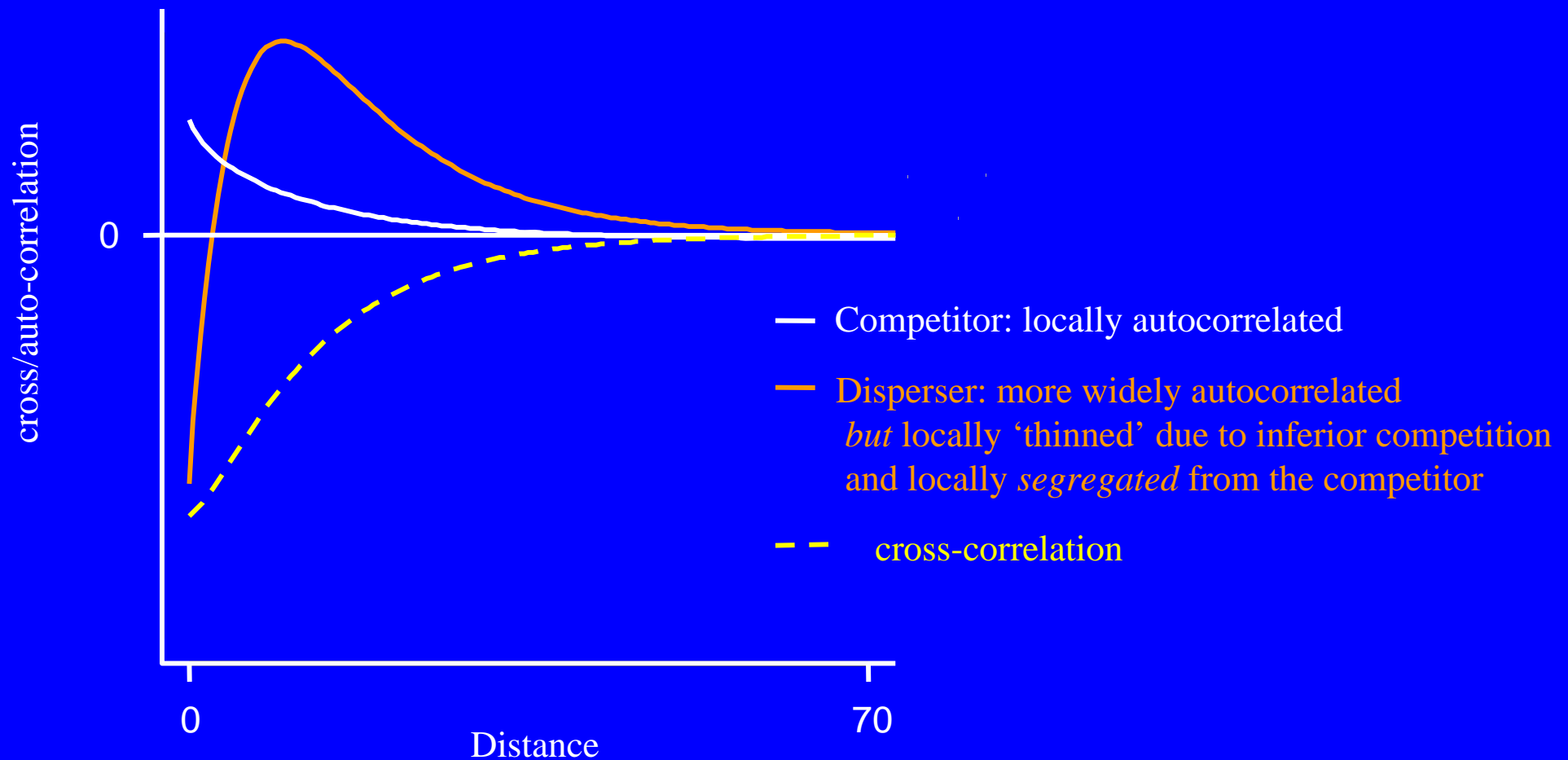
- Local competition (for light and nutrients)
- Seed dispersal

2.



Predicted auto/cross- correlation:

Eg competitively dominant (but weak disperser) vs
mobile (but weak competitor)



Interspecific processes: (2) predation



(2) predation

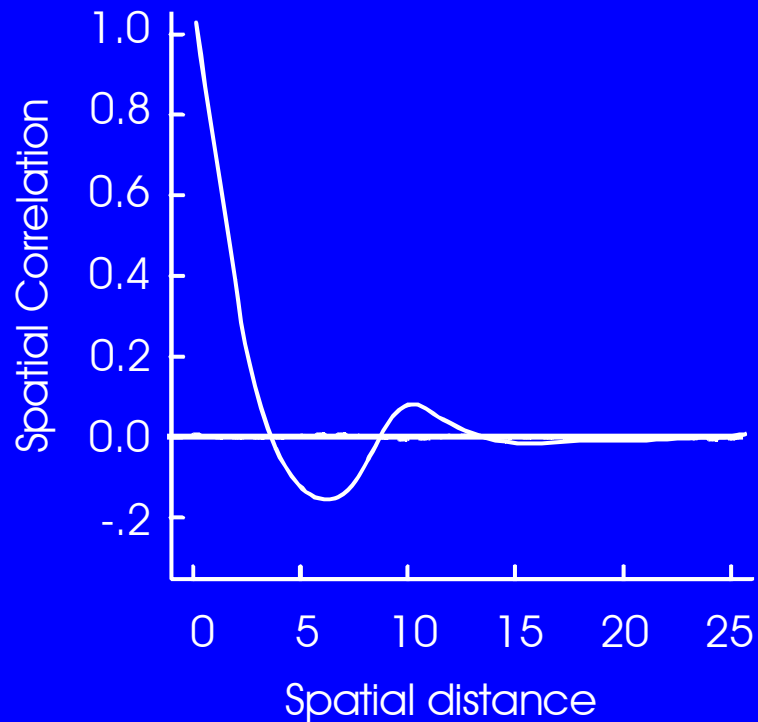


Redistribution

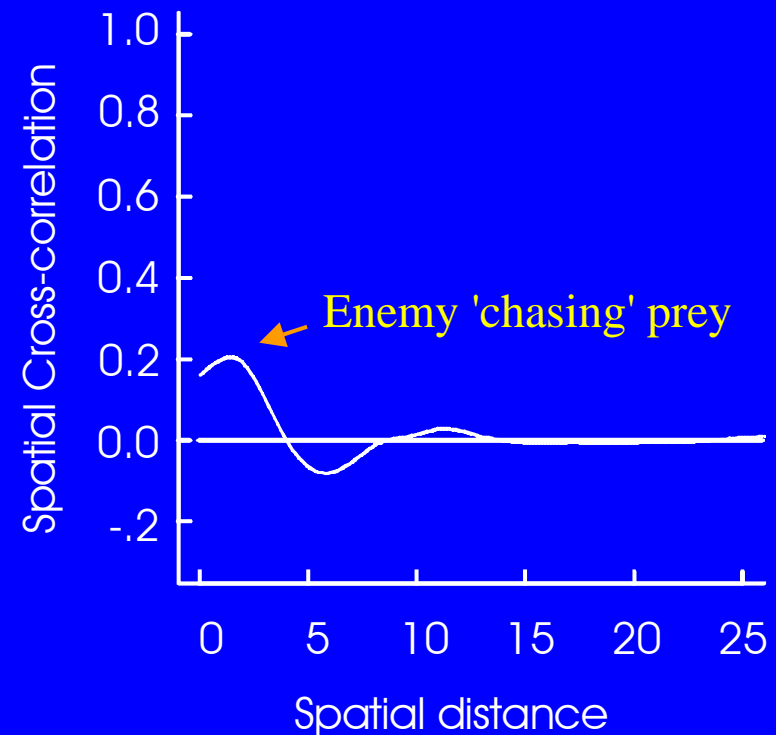


(Cross-)Correlation functions in predator-prey systems

Prey autocorrelation function



Cross-correlation fn



Cyclic correlation functions from traveling waves:
Predator population 'chasing' prey numerically in space & time

Part 2) Statistical estimation

- Geostatistics offer numerous parametric models
(eg Spherical, Exponential or Gaussian models)
but these are not flexible enough

- The classical *nonparametric* method is the correlogram or variogram
but these do not offer estimates of the *correlation functions*

The nonparametric spatial covariance functions

$$\tilde{\rho}(\delta) = \frac{\sum_{i=1}^N \sum_{j=i+1}^N G(\delta_{ij}/h) \rho_{ij}}{\sum_{i=1}^N \sum_{j=i+1}^N G(\delta_{ij}/h)}$$

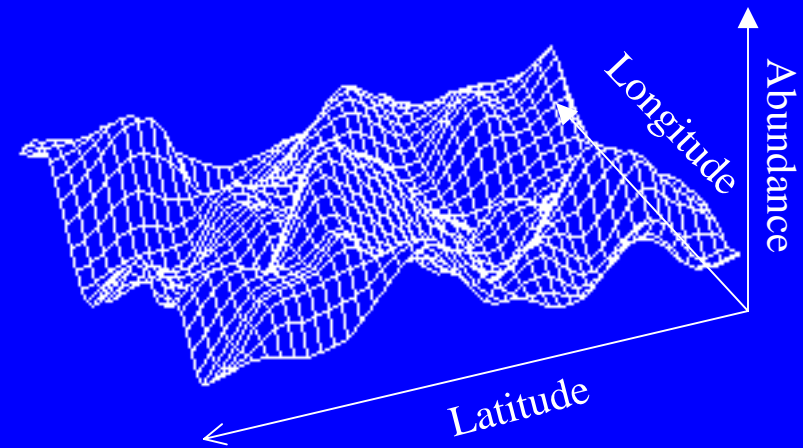
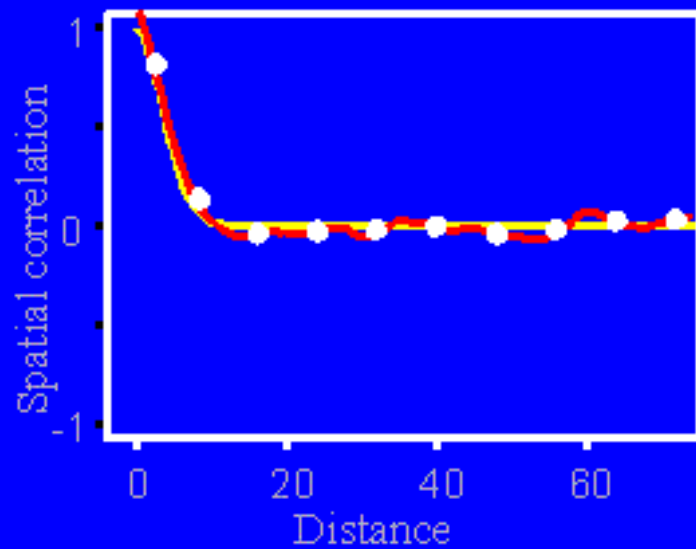
where G is a kernel function with kernel bandwidth h

: A kernel regression of pairwise correlation, ρ_{ij} , on pairwise distance, δ_{ij} .

[But C-fns must obey additional constraints

Beuchners thm: Fourier filtering ensures positive semi-definiteness the resultant function **will be** a permissible correlation function]

NCF – test using simulated data



— True correlation function

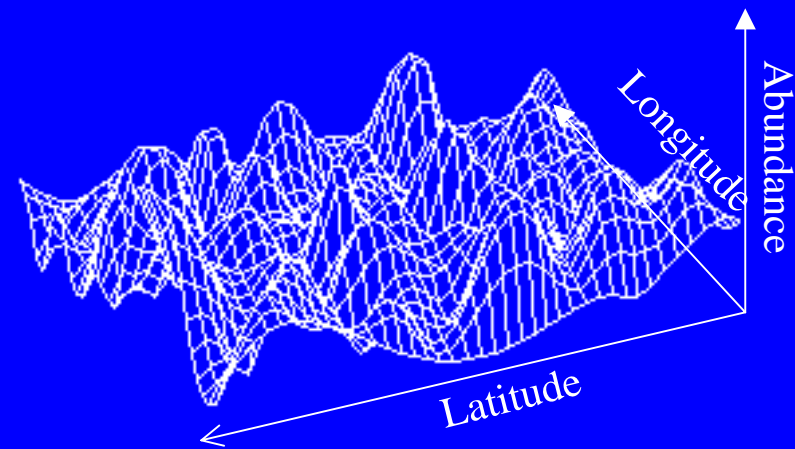
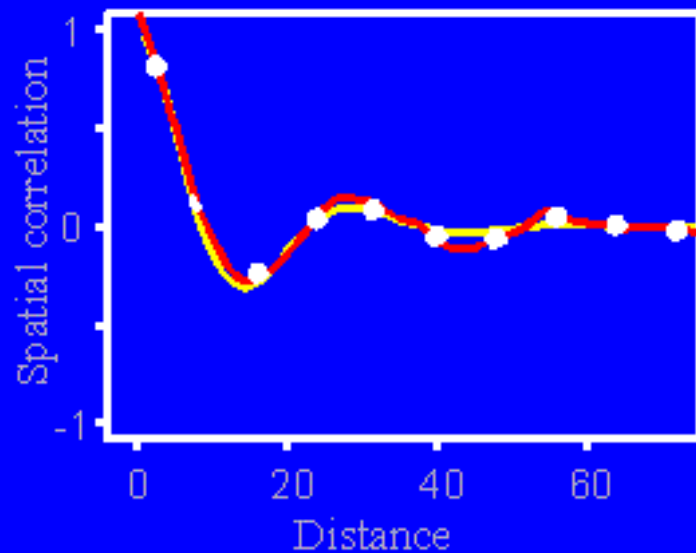
— NCF estimate

(• Spatial correlogram)

Estimation is based on 400 observations from simulated data

NCF – test using simulated data (2)

The nonparametric estimator applied to 'spatially cyclic' data



- True correlation function
- NCF estimate
- (• Spatial correlogram)

Part 3) DATA: spatial grass competition

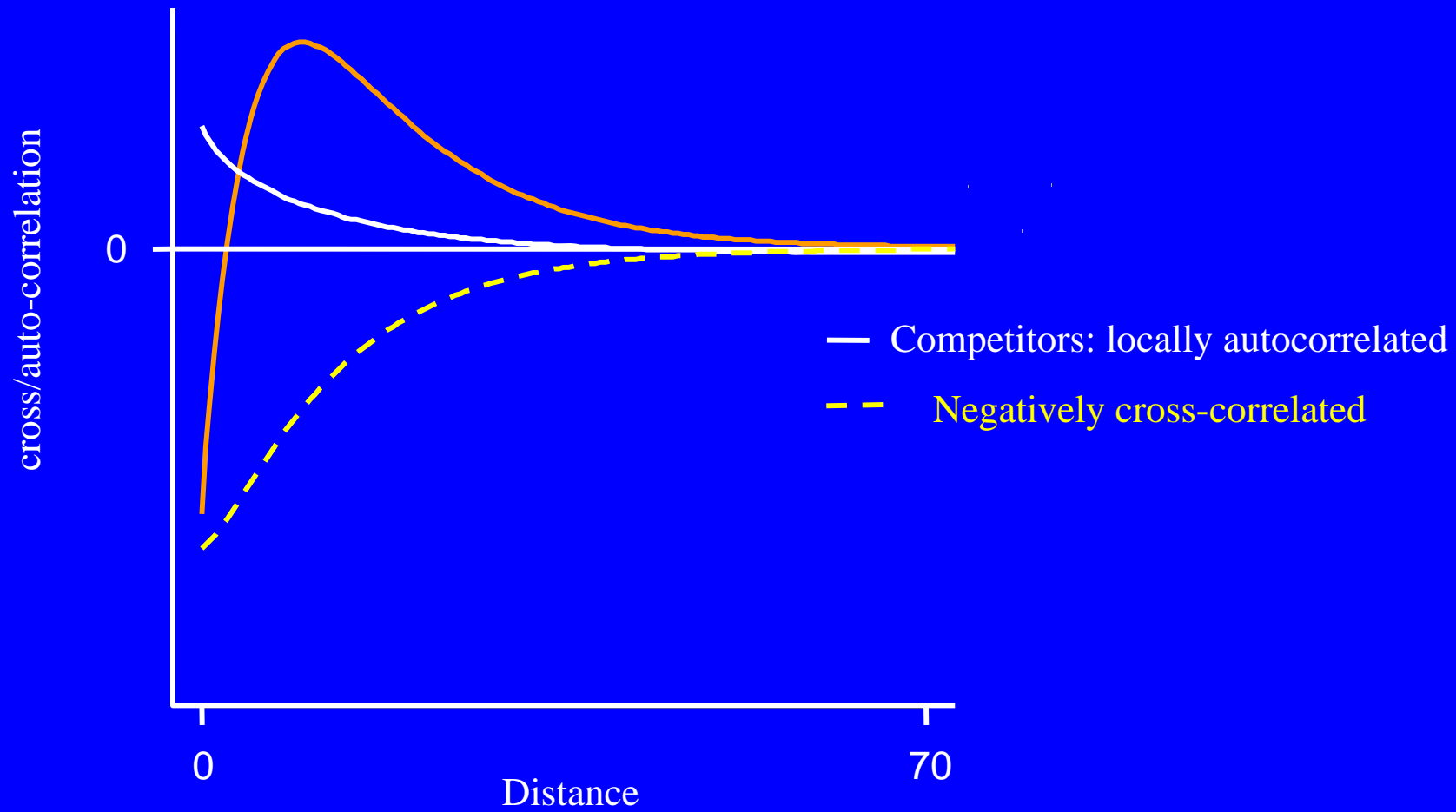


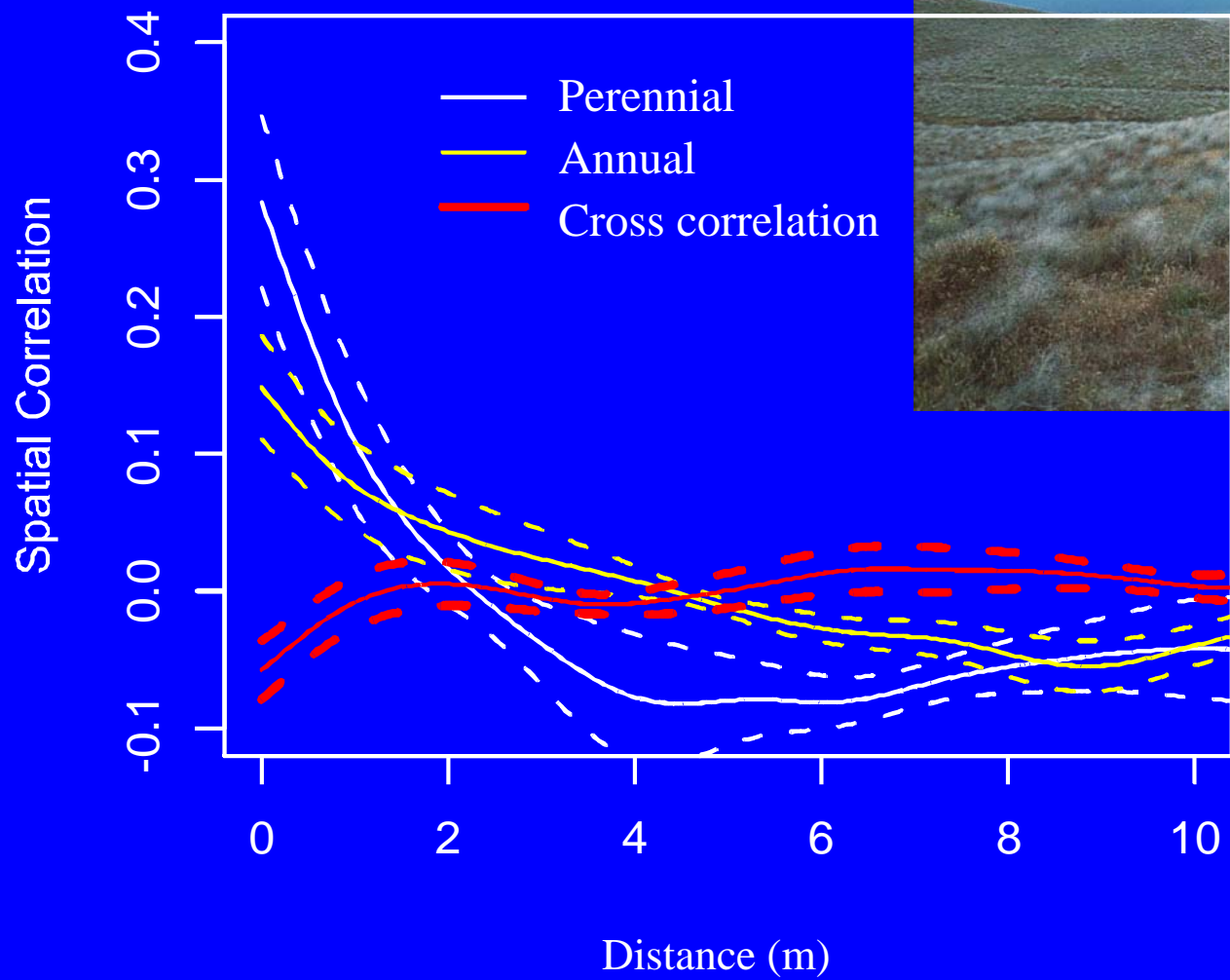
Spatial patterns in experimental grasslands



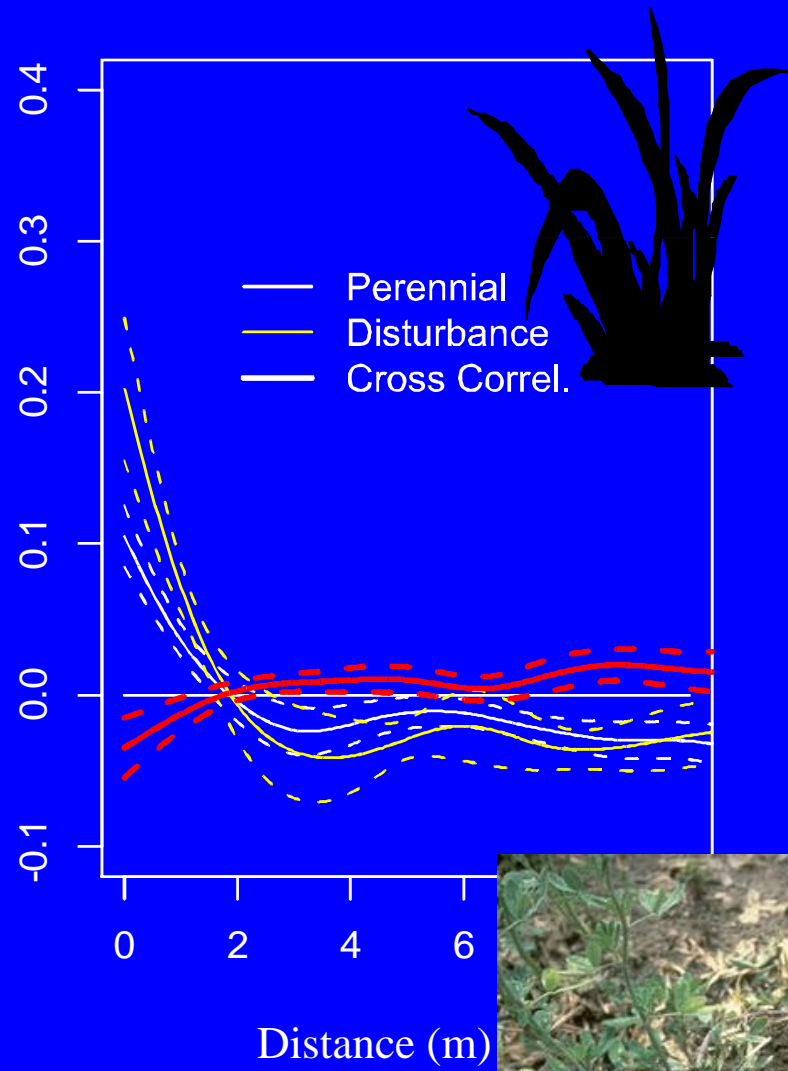
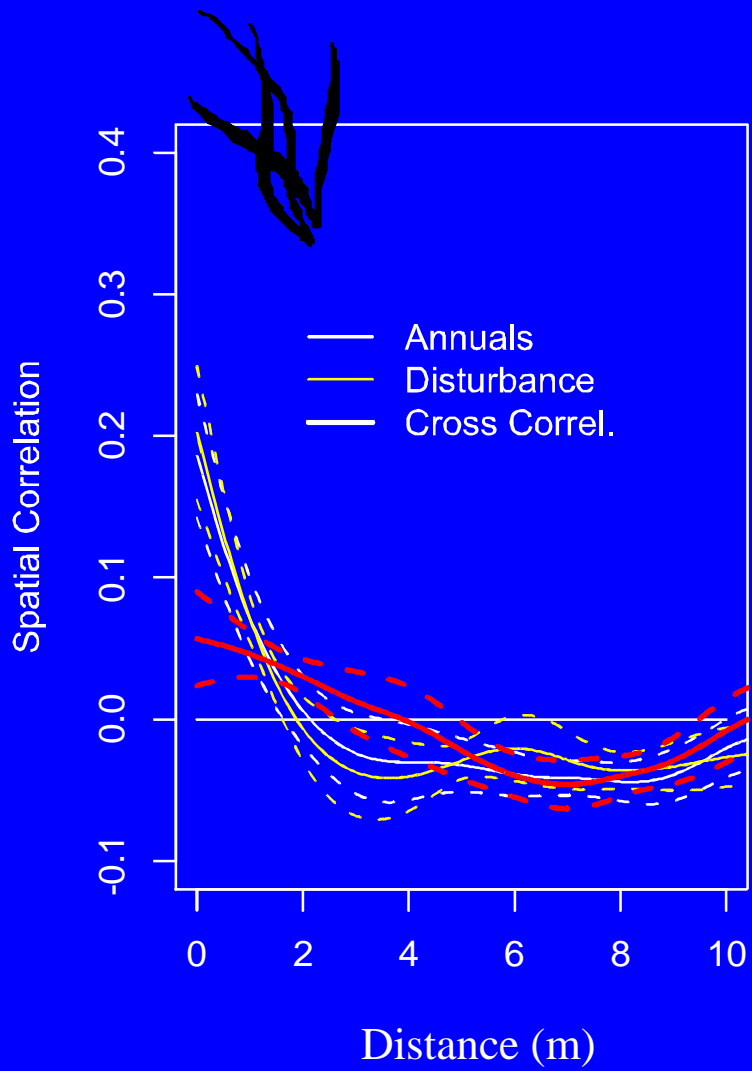
Sedgewick reserve,
S California

Recall prediction form theory:





— Estimate
- - - 95% Bootstrap CI



Conclusions:

- Ecological processes leave detailed spatial signatures
- Spatial autocorrelation and cross-correlation functions is a meeting ground for ecological theory and data
- Moment equations offer detailed predictions
- The nonparametric correlation function offers statistical estimates
- R-package and reprints on
<http://asi23.ent.psu.edu> (Bjornstad)
- Details on Moment equations on
<http://www.zoo.ufl.edu/bolker/> (Bolker)
- Data repository
<http://science.oregonstate.edu/~seabloom/> (Seabloom)