

Eigenvalues and eigenvectors

1. Orthogonal vectors

$$\mathbf{b} \cdot \mathbf{c} = |\mathbf{b}| \cdot |\mathbf{c}| \cdot \cos \theta$$

If $\theta = 90^\circ$, $\cos \theta = 0$, hence $\mathbf{b} \cdot \mathbf{c} = 0$

p. 65

2. Rank of a matrix

$$\text{Rank}(\text{mat}) < \text{size}(\text{mat}) \quad \Rightarrow \quad \det(\text{mat}) = 0$$

p. 72

3. Eigenanalysis

$$\begin{aligned} \mathbf{S} \mathbf{u}_i &= \lambda_i \mathbf{u}_i & \Rightarrow & \mathbf{S} \mathbf{u}_i - \lambda_i \mathbf{u}_i = \mathbf{0} \\ & & \Rightarrow & (\mathbf{S} - \lambda_i \mathbf{I}) \mathbf{u}_i = \mathbf{0} \end{aligned}$$

pp. 80-83

Reasoning:

1) $\mathbf{u}_i = \mathbf{0}$?

2) $(\mathbf{S} - \lambda_i \mathbf{I}) = \mathbf{0}$?

3) $(\mathbf{S} - \lambda_i \mathbf{I})$ is orthogonal to \mathbf{u}_i

Property of orthogonal vectors and matrices

Then the product of $(\mathbf{S} - \lambda_i \mathbf{I})$ by \mathbf{u}_i is a null vector

Numerical solution: find λ_i such that

$$|\mathbf{S} - \lambda_i \mathbf{I}| = 0$$

Property of Rank(mat) < size(mat)

Example pp. 83-85

$$\mathbf{A} = \text{matrix}(\text{c}(2,2,2,5), 2, 2)$$

$$\det(\mathbf{A}) =$$

• Find the eigenvalues

$$\text{eigen.out} = \text{eigen}(\mathbf{A})$$

$$\text{eigen.out}\$values =$$

• Check that $|\mathbf{S} - \lambda_i \mathbf{I}| = 0$ for each λ_i

$$\lambda_1 = 6 \quad \Rightarrow \quad \text{matLambda1} = \text{matrix}(\text{c}(-4,2,2,-1), 2, 2)$$

$$\det(\text{matLambda1}) =$$

$$\lambda_2 = 1 \quad \Rightarrow \quad \text{matLambda2} = \text{matrix}(\text{c}(1,2,2,4), 2, 2)$$

$$\det(\text{matLambda2}) =$$