Eigenvalues and eigenvectors

1. Orthogonal vectors

$\mathbf{b} \bullet \mathbf{c} = |\mathbf{b}| \bullet |\mathbf{c}| \bullet \cos\theta$ If $\theta = 90^\circ$, $\cos\theta = 0$, hence $\mathbf{b} \bullet \mathbf{c} = 0$ *p. 65*

2. Rank of a matrix

Rank(mat) < size(mat) $\Rightarrow$ det(mat) = 0 *p. 72*

3. Eigenanalysis

$\mathbf{S}\,\mathbf{u}_i = \lambda_i\,\mathbf{u}_i \quad \Rightarrow \quad \mathbf{S}\,\mathbf{u}_i - \lambda_i\,\mathbf{u}_i = \mathbf{0}$ *pp. 80-83*

$\Rightarrow \quad (\mathbf{S} - \lambda_i\,\mathbf{I})\,\mathbf{u}_i = \mathbf{0}$

Reasoning:

1) $\mathbf{u}_i = \mathbf{0}$?

2) $(\mathbf{S} - \lambda_i\,\mathbf{I}) = \mathbf{0}$?

3) $(\mathbf{S} - \lambda_i\,\mathbf{I})$ is orthogonal to $\mathbf{u}_i$ *Property of orthogonal vectors and matrices*

Then the product of $(\mathbf{S} - \lambda_i\,\mathbf{I})$ by $\mathbf{u}_i$ is a null vector

Numerical solution: find λ_i such that

$|\,\mathbf{S} - \lambda_i\,\mathbf{I}\,| = 0$ *Property of Rank(mat) < size(mat)*

Example pp. 83-85

A = matrix(c(2,2,2,5), 2, 2) det(**A**) =

• Find the eigenvalues

eigen.out = eigen(A) eigen.out$values =

• Check that $|\,\mathbf{S} - \lambda_i\,\mathbf{I}\,| = 0$ for each λ_i

$\lambda_1 = 6 \quad \Rightarrow$ matLambda1 = matrix(c(-4,2,2,-1), 2, 2) det(matLambda1) =

$\lambda_2 = 1 \quad \Rightarrow$ matLambda2 = matrix(c(1,2,2,4), 2, 2) det(matLambda2) =