Multiple regression:

Computing the parameter estimates by matrix inversion

Data table:

$\mathbf{y}$	$\mathbf{x}_1$	$\mathbf{x}_2$	-----	$\mathbf{x}_m$
y_1	x_{11}	x_{12}	-----	x_{1m}
y_2	x_{21}	x_{23}	-----	x_{2m}
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y_n	x_{n1}	x_{n4}	-----	x_{nm}

Matrix notation for the regression equation: $\mathbf{y} = \mathbf{X}\,\mathbf{b}$

$$\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ \dots \\ y_n \end{bmatrix} \quad \mathbf{X} = \begin{bmatrix} 1 & x_{11} & x_{12} & \dots & x_{1m} \\ 1 & x_{21} & x_{22} & \dots & x_{2m} \\ \dots & \dots & \dots & \dots & \dots \\ 1 & x_{n1} & x_{n2} & \dots & x_{nm} \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ \dots \\ b_m \end{bmatrix}$$

Adding a column of “1” to matrix **X** will allow the estimation of an intercept b_0.

Reasoning (*Numerical ecology* 1998, p. 79) :

$\mathbf{y} = \mathbf{X}\ \mathbf{b}$

$\mathbf{X'y} = \mathbf{X'X}\ \mathbf{b} \quad \Rightarrow \quad [\mathbf{X'X}]^{-1}[\mathbf{X'y}] = [\mathbf{X'X}]^{-1}\ [\mathbf{X'X}]\ \mathbf{b}$

$\Rightarrow \quad [\mathbf{X'X}]^{-1}[\mathbf{X'y}] = \mathbf{b}$

Computing the fitted values of the regression equation:

$$\hat{\mathbf{y}} = \mathbf{X}\mathbf{b}$$

$$\hat{\mathbf{y}} = \mathbf{X}\,[\mathbf{X'X}]^{-1}\mathbf{X'}\ \mathbf{y}$$

$$\hat{\mathbf{y}} = \boxed{\mathbf{X}\,[\mathbf{X'X}]^{-1}\mathbf{X'}}\ \mathbf{y}$$

Projector

During a permutation test, the projector does not have to be re-computed after each permutation of the **y** data. One can simply compute:

$$\hat{\mathbf{y}}_{\text{permuted}} = \boxed{\mathbf{X}\,[\mathbf{X'X}]^{-1}\mathbf{X'}}\ \mathbf{y}_{\text{permuted}}$$

$$\hat{\mathbf{y}}_{\text{permuted}} = \textbf{Projector}\ \mathbf{y}_{\text{permuted}}$$